XXIV. On the Structure of the Atom: an Investigation of the Stability and Periods of Oscillation of a number of Corpuscles arranged at equal intervals around the Circumference of a Circle; with Application of the Results to the Theory of Atomic Structure

by J.J. Thomson, F.R.S., Cavendish Professor of Experimental Physics, Cambridge

Philosophical Magazine
Series 6, Volume 7, Number 39
March 1904, p. 237-265

The view that the atoms of the elements consist of a number of negatively electrified corpuscles enclosed in a sphere of uniform positive electrification, suggests, among other interesting mathematical problems, the one discussed in this paper, that of the motion of a ring of \( n \) negatively electrified particles placed inside a uniformly electrified sphere. Suppose when in equilibrium the \( n \) corpuscles are arranged at equal angular intervals round the circumference of a circle of radius \( a \), each corpuscle carrying a charge \( e \) of negative electricity. Let the charge of positive electricity be \( ne \), then if \( b \) is the radius of this sphere, the radial attraction on a corpuscle due to positive electrification is equal to \( ne^2a/b^2 \); if the corpuscles are at rest this attraction must be balanced by the repulsion exerted by the other corpuscles. Now, the repulsion along OA, O being the centre of the sphere, exerted on a corpuscle at A by one at B is equal to \( (e^2 / AB^2) \cos OAB \) and, if \( OA = OB \), this is equal to \( e^2 / (4 OA^2 \sin 1/2 AOB) \); hence, if we have \( n \) corpuscles arranged at equal angular intervals \( 2\pi / n \) round the circumference of a circle, the radial repulsion on one corpuscle due to the other \( (n - 1) \) is equal to . . .

[Thomson then plunges into many pages of computations on the stable arrangements of corpuscles within the atom.]

resuming on p. 255 . . .

Application of the preceding Results to the Theory of the Structure of the Atom
We suppose that the atom consists of a number of corpuscles moving about in a sphere of uniform positive electrification: the problems we have to solve are (1) what would be the structure of such an atom, \textit{i.e.} how would the corpuscles arrange themselves in the sphere; and (2) what properties would this structure confer upon the atom. The solution of (1) when the corpuscles are constrained to move in one plane is indicated by the results we have just obtained -- the corpuscles will arrange themselves in a series of concentric rings. This arrangement is necessitated by the fact that a large number of corpuscles cannot be in stable equilibrium when arranged as a single ring, while this ring can be made stable by placing inside it an appropriate number of corpuscles. When the corpuscles are not constrained to one plane, but can move about in all directions, they will arrange themselves in a series of concentric shells; for we can easily see that, as in the case of the ring, a number of corpuscles distributed over the surface of a shell will not be in stable equilibrium if the number of corpuscles is large, unless there are other corpuscles inside the shell, while the equilibrium can be made stable by introducing within the shell an appropriate number of other corpuscles.

The analytical and geometrical difficulties of the problem of the distribution of the corpuscles when they are arranged in shells are much greater than when they are arranged in rings, and I have not as yet succeeded in getting a general solution. We can see, however, that the same kinds of properties will be associated with the shells as with the rings; and as our solution of the latter case enables us to give definite results, I shall confine myself to this case, and endeavour to show that the properties conferred on the atom by this ring structure are analogous in many respects to those possessed by the atoms of the chemical elements, and that in particular the properties of the atom will depend upon its atomic weight in a way very analogous to that expressed by the periodic law.

Let us suppose, then, that we have \( N \) corpuscles each carrying a charge \( e \) of negative electricity, placed in a sphere of positive electrification, the whole charge in the sphere being equal to \( Ne \); let us find the distribution of the corpuscles when they are arranged in what we may consider to be the simplest way, \textit{i.e.} when the number of rings is a minimum, so that in each ring there are as nearly as possible as many corpuscles as it is possible for the corpuscles inside to hold in equilibrium. Let us suppose that the number of internal corpuscles required to make the equilibrium of a ring of \( n \) corpuscles stable is \( f(n) \). The values of \( f(n) \) for a series of values of \( n \) is given in the table on p. 254; in that table \( f(n) \) is denoted by \( p \). The number of corpuscles in the outer ring \( n_1 \) will then be determined by the condition at \( N - n_1 \), the number of
corpuscles inside, must be just sufficient to keep the ring of \( n_1 \) corpuscles in equilibrium, i.e., \( n_1 \) will be determined by the equation

\[ N - n_1 = f(n_1) \]

If the value of \( n_1 \) got from this equation is not an integer we must take the integral part of the value. [The paper continues to p. 265. Someday. . . .]